Multicriterial optimisation of squirrel-cage induction motor design

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Abstract: The task of finding an optimal induction-motor design is defined as a discrete multicriterial optimisation problem. An approach to solving this problem through two-level optimisation is described. A bicriterial problem is solved by the assumption that all optimisation variables are continuous. Criterial functions in the example are material and operating costs of the motor. The possibility of applying two methods (multipliers and constraints) is considered and both these methods are employed in the optimisation process. Corresponding scalar optimisation problems are solved by means of a quadratic approximation method. An analysis of obtained optimisation results is performed, with the major conclusions that two local compromise solution sets exist and that the global compromise set in the decision space is not continuous. The computational model (more sophisticated than in the majority of other works dealing with the optimisation of an induction-motor design) and the synthesis program are briefly described. The results of design calculations are analysed mainly for two optimal solutions determined for minimal operating and material costs, although intermediate results are also presented.

1 Introduction

Perhaps the first work concerning an application of optimisation to an induction-motor design appeared at the end of the 1950s, not long after computers were introduced to civil engineering [1]. At that time, the low efficiency of computers and optimisation procedures involved considerable simplifications of the motor models and computer programs. Today, optimisation methods are commonly used in the design process. In general, a design process may be considered to be the result of some compromise decisions related to accuracy requirements and calculations costs. Thus, computer equipment is usually an important factor in this process.

In calculations, optimisation variables are usually continuous and an objective function is scalar [1–6]. In fact, almost all design parameters are discrete for physical, manufacturing or economical reasons. Often some parameters, for instance stator and rotor slots numbers, are predetermined by a designer and remain constant in an optimisation process. Afterwards, if necessary, the whole procedure is repeated for another set of values of the above parameters. Other parameters, like conductors or slots dimensions, can be assumed to be continuous [3, 5, 6] at the first stage of calculations. However, if the desired discrete values are determined from the solution of a continuous optimisation problem through a round-off process, then the consequence of such an approach is that imposed constraints are usually violated and the accepted design solution is no longer optimal. Another approach to solve design problems with discrete variables is applying problem-orientated methods. The description of such an attempt can be found in Reference 7.

The past decade has witnessed a growing number of works devoted to the application of multicriterial optimisation to design problems. Reference 8 may be considered as an example of employing the above approach when designing an induction motor. A relatively simple model of the motor has been used in that work. In this paper calculations are performed for a much more sophisticated computational model, which increases the complexity of the resulting computer program but reduces the level of uncertainty in the optimisation results.

The most frequently used optimisation criterial functions are material or production costs and the operating costs of the motor. If the objective function represents the manufacturer's viewpoint, then it is equal to the material costs [1, 3–5, 9–11]. Other production costs are usually assumed to be constant in an optimisation process. The objective function, in the form of a weighted sum of the material and operating costs, represents the consumer's viewpoint [2, 6, 9, 11]. The minima of material and operating costs are usually conflicting criteria in the area of practical interest and corresponding cost functions may be used in a multicriterial optimisation approach.

A set of analytical formulas, curves, tables and iterative procedures that enable the designer to perform all the necessary design calculations will be called a computational model. The result of these design calculations will be called a design solution, if it depends on a predetermined set of design variables. In an optimisation process these design variables are called optimisation or decision variables. A synthesis program will be a computer program performing design calculations according to an algorithm defined in the computational model, and providing, as a result of calculations of a design solution in the form of performance characteristics, values of an assumed objective function and constraints.

The search for an optimal design solution of a squirrel-cage induction motor will be described in the paper as a multicriterial discrete, nonlinear optimisation problem, but calculations will be performed on the assumption that optimisation variables are only those

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which can be considered as continuous. Some remarks, calculations results and conclusions related to the solved problems will be presented.

The example of calculations in this paper may be considered as an extension of the economical analysis presented in Reference 9 in both the computational model and the research program.

The corresponding full-load values are the reference units for all p.u. values in Figs. 6 and 7 and in constraints descriptions.

2 Induction-motor design as an multicriterial discrete optimisation problem

Let $x = [x_1, x_2, \ldots, x_{n_c}]^T$ be the vector of $n_c \geqslant 1$ continuous decision variables, $y = [y_1, y_2, \ldots, y_{n_d}]^T$ be the vector of $n_d \geqslant 1$ discrete decision variables, $z = [z_1, z_2, \ldots, z_K]^T$ be the vector of $K \geqslant 2$ indexes to be minimised. The components of x and y correspond to the design parameters assumed to be the optimisation variables. The indexes z_k for $k = 1, 2, \ldots, K$ are defined by means of independent optimisation criterial functions $z_k = f_k(x, y) \in R^1$. The functions f_k are nonlinear and unique in the case of an induction-motor design.

The multicriterial discrete optimisation problem related to a design may be described in general form as

Problem P_1:

$$\min_{x, y} f_1(x, y), f_2(x, y), \ldots, f_K(x, y) \Big|_{x \in X(y),\, y \in Y} \qquad (1)$$

where the set

$$X(y) = \{x : g_j(x, y) \leqslant 0, h_l(x, y) = 0, \\ x \in R^{n_c}, y \in R^{n_d}, j = 1, 2, \ldots, m_i, l = 1, 2, \ldots, m_e\}$$

describes a feasible region for x, and the set Y is an assumed feasible region for y. Functions g_j and h_l are nonlinear or linear real functions. The set X is dependent on the vector y. Both the sets X and Y result from standards, physical and technological restrictions, performance requirements, design assumptions etc. The set Z of all attainable index vectors z satisfying imposed constraints can be defined as

$$Z = \{z : z_k = f_k(x, y), x \in X(y), y \in Y, k = 1, 2, \ldots, K\}$$

If the criteria represented by $f_1, f_2, \ldots, f_K$ are not all cooperative, then the solution of Problem P_1 is not unique. In the literature one can find the following names of this solution: Pareto-optimal, supremal, efficient, noninferior etc. For example, a feasible solution $[x^*, y^*]$ will be called Pareto-optimal if no $[x, y]$ exist such that $f_i(x, y) \leqslant f_i(x^*, y^*)$ for all i and $f_k(x, y) < f_k(x^*, y^*)$ for at least one k. Similarly, an index vector $z^* \in Z$ is called [12–14] supremal in relation to any $z \in Z$ if $z_i^* \leqslant z_i$ for all i and $z_k^* < z_k$ for at least one k. The set $\{[x^*, y^*]\}$ in the decision space and the set $\{z^*\}$ in the objective space are the sets of the best solutions in the sense of the above definitions. In this paper these sets will be called compromise solutions sets or simply compromise sets, in a corresponding space. When solving design optimisation problems the determination of both these sets is usually a simultaneous process, meaning that the analysis of the solution of Problem P_1 in the objective space is sufficient to determine a compromise set $S = \{z^*\}$ in this space, as well as a corresponding set $P = \{[x^*, y^*]\}$ in the decision space.

The existence of discrete variables is a reason that the functions f_k, g_j, h_l are not continuous and the sets X, Y, Z may neither be convex nor even connected. The solution of Problem P_1 in such a form as in eqn. 1 may appear to be difficult. A reasonable approach to solve this problem seems to be to apply the well known decomposition method. In this case such a method allows the designer to present Problem P_1 in the form of a two-level optimisation problem.

Problem P_2:

$$\min_{y} z^*(y) \Big| y \in Y, z^*(y) = \min_{x} z(x, y) \Big|_{x \in X(y)} \qquad (2)$$

The determination of the vector z^* for a given y is the task of the first level and the search for the minimum of z^* is the task of the second optimisation level. Solving Problem P_2 is usually much simpler than solving Problem P_1, especially when the number of the elements of Y is small.

The two best known methods to determine the compromise sets S and P are the multipliers or weighting method and the constraints method. In both methods a similar computational effort must be made and both methods provide a decision maker with the maximum quantity of desirable information. The most frequently used multipliers method seems to be simpler in application but the efficiency of this method depends on the condition of the directional convexity of Z [12]. This means that the multipliers method can be applied at most at the first optimisation level of Problem P_2. In this method a synthetic objective function F in the form

$$F = \sum_{k=1}^{K} \lambda_k f_k(x, y),\ R^1 \ni \lambda_k \geqslant 0,\ \sum_{k=1}^{K} \lambda_k = 1 \qquad (3)$$

is introduced and the task of the first optimisation level is replaced by a sequence of scalar optimisation problems.

Problem P_3:

$$\min_{x} F(x, y, \lambda) \Big|_{x \in X(y),\, y \in Y,\, \lambda = [\lambda_1, \lambda_2, \ldots, \lambda_K]^T} \qquad (4)$$

for subsequent multipliers sets $\{\lambda_k\}$ satisfying eqn. 3. In general, every element of the set S is a solution of Problem P_3, although the reverse is not true. If criterial functions f_k are not commensurable a scaling procedure is commonly used.

A modification of the constraints method does not require any assumption related to the convexity of Z. The description of the proper equality constraints method (PEC) and the proper inequality constraints method (PIC) can be found in References 13 and 14. The PIC method applied at the first optimisation level requires the solutions and their analysis of the following scalar optimisation problems.

Problem P_4:

$$\min_{x} f_j(x, y) \Big|_{f_k \leqslant \alpha_k,\, k \neq j,\, x \in X(y),\, y \in Y} \qquad (5)$$

for any $j \in \{1, 2, \ldots, K\}$ and every proper set $\{\alpha_k\}$. The parameters α_k, $k = 1, 2, \ldots, K$, $k \neq j$, must be properly chosen real numbers. The PIC and PEC methods are more general than the multipliers; the solutions of

Problem P_4 defined for both the vectors x and y can be used when searching for the solutions of Problem P_1 in its original form eqn. 1.

3 Scalar optimization

Compromise sets S and P mentioned in the previous Section consist of the solutions of proper scalar optimisation problems. Even in the scalar case the search for an optimal design solution of an induction motor is a complex problem of nonlinear programming.

A few attempts have been made by different authors to present comparative conclusions relating to the efficiency and reliability of existing optimisation programs. Useful information concerning such conclusions and further details can be found, for instance, in References 15–17 dealing with general test examples, or in References 10 and 18 relating to the particular case of an induction-motor design.

To a high degree the results of design optimisation computations depend on the quality of the synthesis program and they are not only influenced by the properties of the physical and mathematical models of the motor. Thus, a number of numerical experiments should be performed before a decision is made as to what optimisation procedure to apply. A possibility in the example below of applying more than ten such procedures and resulting computer programs, described in References 19–21, has been investigated. The results of the corresponding analysis will be presented in the future.

4 Physical and computational models

The squirrel-cage induction motor analysed here has been characterised by lumped parameters. It means that the physical description of the motor is reduced to equivalent parameters such as resistances and inductances, and the final representation is in the form of the well known induction-motor equivalent circuit. The question of how to derive the values of the circuit parameters to ensure the requested accuracy of this model is still under discussion, mainly because the validity of such models is limited. Perhaps models describing the motor as a distributed-parameters system and arising from corresponding electromagnetic field equations will make satisfactory progress in the matter.

The design calculations procedure applied in the computational model is based on Reference 22, but necessary modifications had to be carried out in order to adopt it to optimisation purposes. Performance characteristics of the motor are derived from the equivalent circuit presented in Fig. 1 in two forms. Formulas calculating stator and rotor leakage reactances are derived on the assumption that the magnetic field in slots is rectilinear. The well known skin effect parameters k_R and k_X represent an influence of skin effect on rotor parameters values. Norman's curve is used to take into account the influence of the saturation on stator and rotor leakage reactances. In the case of short-circuit calculations a demagnetisation of the magnetic circuit is taken into account, as is suggested in Reference 22. The total active power losses comprise stator and rotor I^2R losses, mechanical losses, additional losses under load and iron losses, which consist of main, surface and tooth-pulsation losses. Formulas for the losses, temperature rise, airflow, insulation specification, load and no-load conditions are taken from Reference 22.

The procedure described in Reference 23 is employed to obtain the magnetising reactance, the saturation factor k_μ and to perform the whole magnetic calculations.

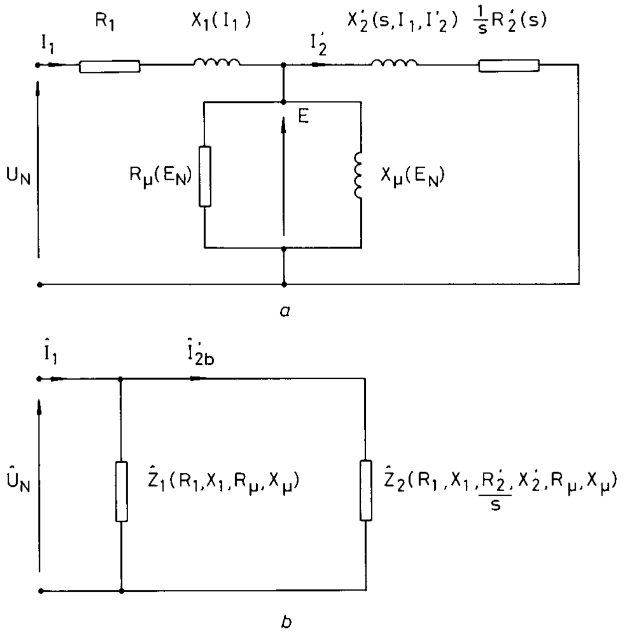


Fig. 1 *Representations of equivalent circuit used in computational model*
a Classical
b Converted (in complex form)

The algorithm in the computational model is determined so that, according to the standards [24], the core losses correspond to the full-load condition. Other requirements of these standards concerning the designed motor, chiefly related to additional load losses, are also satisfied.

All performance characteristics of the motor are described in an implicit form and are determined through multiloop iterative calculations.

5 Synthesis program

It is a time-consuming task to work out a good quality synthesis program satisfying all conditions imposed by the assumed computational model and accuracy requirements of optimisation calculations. One of the most important questions is how to obtain an efficient solution of a few highly nonlinear, simultaneous algebraic equations on the assumption that some relationships are in the form of curves or tables. Such an efficient solution is the result of calculations obtained as fast and as accurately as possible. For example, to perform magnetic calculations the program solves a nonlinear equation, but four such simultaneous equations should be solved to perform accurate calculations for any load condition. Maximum torque is determined through a local one-dimensional optimisation procedure. More details are given in the example below.

All necessary calculations in the synthesis program are completed with accuracy level allowing an application of gradient optimisation methods. There are two reasons to do so. First, gradient methods usually appear to be more efficient than direct-search methods; such a property is important in the case of the multicriterial optimisation.

Secondly, the knowledge of the gradients and Hessian of the assumed objective function and constraints enables the designer to perform a detailed analysis in the vicinity of the optimal point.

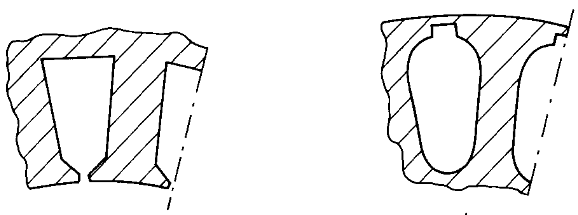
6 Example

The example concerns the application of bicriterial optimisation to induction-motor designing. Calculations are performed at the first optimisation level of Problem P_2. The compromise solution set S estimated in the objective space make-up solution of problems P_3 and P_4.

6.1 Assumptions

Under consideration is an induction motor described as a design calculations example in Reference 22. The motor is a three-phase, squirrel-cage induction motor with the following data: $P_N = 15$ kW, $U_N = 380$ V, star connection, $2p = 4$ poles, $f = 50$ Hz, insulation F, protection $IP44$, continuous duty, totally enclosed and fan-cooled.

The following have been assumed: single-layer stator winding, $n_s = 48$ stator slots, $n_r = 38$ rotor slots, outer stator stack diameter $D_0 = 0.272$ m, shaft height $H = 0.16$ m, the number of parallel stator coils per phase $a = 2$ and the number of wires in a stator conductor $n_{el} = 2$. The rotor cage is made of aluminium and the frame of cast iron. The stator and rotor laminated cores are constructed with EP26 electrical steel with specific losses being 2.4 W/kg. The stator and the rotor slots are sketched in Fig. 2.



a b

Fig. 2 *Shapes of stator and rotor slots and teeth*
a Stator
b Rotor

6.2 Independent variables

The volume of calculations is a reason that in this example the set Y consists only of one element. The assumed above values of n_s, n_r, a and n_{el} are the components of the corresponding vector y of discrete variables.

There are $n_c = 13$ continuous optimisation variables

x_1 = stator bore diameter
x_2 = stator stack length
x_3 = stator tooth height
x_4 = stator tooth width
x_5 = stator tooth opening width
x_6 = rotor tooth height
x_7 = rotor tooth width
x_8 = end-ring width
x_9 = end-ring height
x_{10} = airgap length
x_{11} = stator wire diameter
x_{12} = number of conductors in a stator slot
x_{13} = a ratio k_E of full-load EMF E_N in stator phase winding to the rated stator phase voltage U_N

6.3 Criteria

It was assumed that $K = 2$ critical functions in the form of material costs $f_1 = C_m$ and total operating costs $f_2 = C_o$. The material costs C_m are

$$C_m = m_{Cu}c_{Cu} + m_{Al}c_{Al} + m_{Fe}c_{Fe} + m_f c_f \qquad (6)$$

comprising a stator-winding copper cost (subscript Cu), a rotor-cage aluminium cost (Al), a stator and rotor laminated-core cost (Fe) and a frame cast-iron cost (f). The mass of a material is denoted in eqn. (6) as m and the corresponding price as c.

The operating costs function C_o is assumed in the form

$$C_o = P_N\left[a_1\left(\frac{1}{\eta_l} - 1\right) + a_2\frac{tg\phi_l}{\eta_l}\right] \qquad (7)$$

The function C_o comprises the cost of an active power loss and a reactive energy reduced to the motor installation year. The efficiency η_l and power factor $\cos\phi_l$ correspond to a motor load $P_l = \kappa P_r$, $0 \leqslant \kappa \leqslant 1$. The constant coefficients a_1 and a_2 are

$$a_1 = \frac{\kappa h}{d}, \quad a_2 = \frac{\kappa}{d}[a_q h c_q + (1 - a_q)d_c a_c c_c] \qquad (8)$$

If the discount rate is $s_d = 8\%$ and the amortisation period $\tau = 5$ years, then the amortisation rate is $d = q^\tau(q - 1)/(q^\tau - 1) \cong 0.25$, where $q = 1 + s_d/100$. It is assumed in the calculations that the yearly operating time of the motor is $h = 1600$ h and $P_l = 0.75P_N$. All prices are related to the price of the 1 kW/h and correspond to price levels in Poland in 1985. The unit of relative material prices is kW/h/kg, meaning that the unit of the costs C_m and C_o is kW/h.

The meaning of the other parameters in eqn. 8 is a_q = not compensated part of reactive energy ($a_q \leqslant 1$), c_q = the relative price of 1 kvar/h, d_c = the amortisation rate of a capacitor installation, a_c = the demand factor of a motor group compensated by this installation and c_c = the relative price of installed capacitors (1 kvar). Further details can be found in References 25 and 6. The values of relative prices are assumed in the example as $c_{Cu} = 105 + 0.015/x_{11}$, $c_{al} = 71$, $c_{Fe} = 45$, $c_f = 31$, $c_q = 0.3$, $c_c = 190$. Other parameters are $a_q = 0.1$, $d_c = 0.15$ and $a_c = 0.5$. Finally $a_1 = 4791$ h and $a_2 \cong 182$ h.

6.4 Feasible region

The set X is defined through ten nonlinear inequality constraints, one nonlinear equality constraint and 16 linear inequality constraints. The assumed nonlinear constraints are

g_1^* = p.u. starting torque $\geqslant 1.5$
g_2^* = p.u. starting current $\leqslant 6.5$
g_3 = p.u. maximum torque $\geqslant 2.0$
g_4^* = p.u. no-load stator current $\leqslant 0.4$
g_5^* = full-load slip $\leqslant 0.03$
g_6^* = full-load average stator-winding temperature rise $\leqslant 90$°C
$g_7^* = x_{10} \geqslant 0.00318 - 0.00665/(x_1 + 2.29)$ [11]
g_8^* = stator slot fill factor $\leqslant 0.72$
g_9^* = the ratio of bar to ring current densities $\leqslant 1.3$
$g_{10}^* = g_9^* \geqslant 1.1$
h_1 = the difference between assumed and calculated full-load stator-winding EMF

The above relationships are transformed in the synthesis program into the form presented in the definition of the set X in Section 2. Linear constraints result from physical, technological and other requirements.

6.5 Synthesis program, optimisation procedure

The synthesis program, performing calculations of the assumed objective function, and constraints comprise about 20 subroutines with a total number of approximately 2000 active statements. The general scheme of this program is presented in Fig. 3.

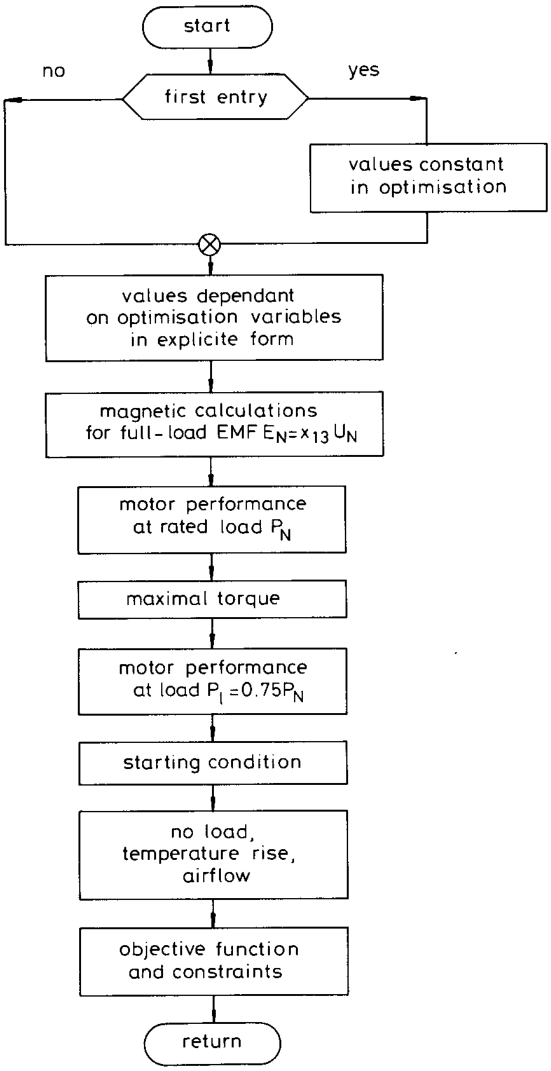


Fig. 3 *General scheme of synthesis program*

Only the values of the most elementary parameters are read in or are stored inside the program. Other constants are calculated by the program. It is possible to choose in the program any subset of the defined maximum set of optimisation variables $\{x_i\}$ and constraints $\{g_j\}$. A suitable combination of the following criterial functions $f_1 = C_m$, $f_2 = C_o$, $f_3 = \eta_l$, $f_4 = \cos \phi_l$ can be chosen to determine a compromise set S comprising solutions of Problems P_3 or P_4.

It has been assumed in the example that the values of magnetising reactance and resistance in the equivalent circuit correspond to the full-load condition, at any load of the motor. This reduces the number of simultaneous algebraic equations when solving the equivalent circuit at a load different from the rated one. The further reduction is possible when the equivalent circuit presented in Fig. 1 in form b is solved. Fig. 4 illustrates the effect of this reduction on the calculation process of the motor performance at load $P_l = 0.75\,P_N$.

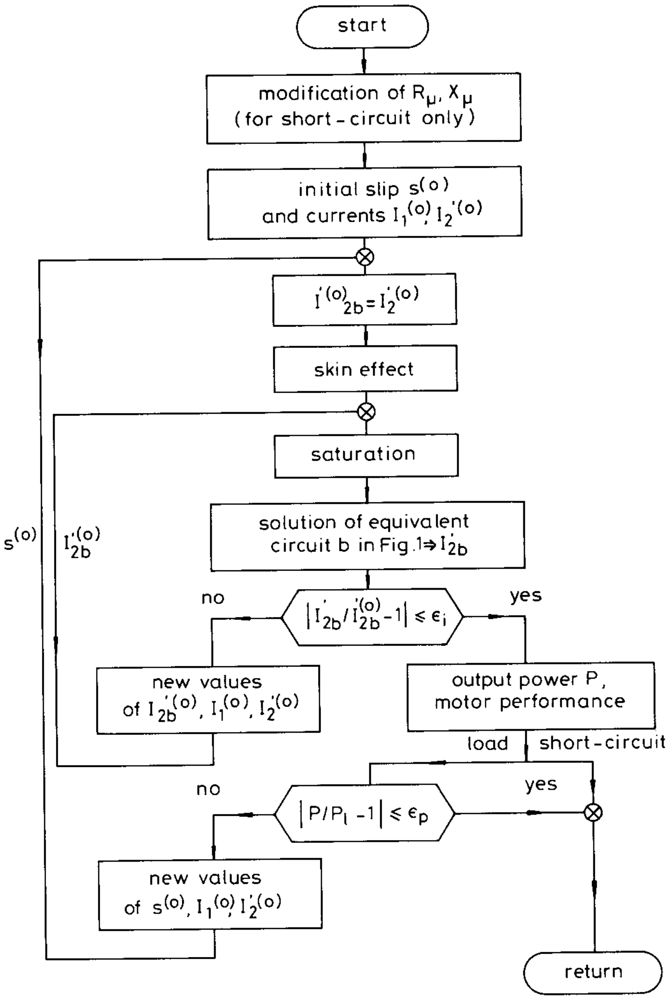


Fig. 4 *Flow diagram of motor performance and starting condition calculations*
Load P_l

The efficiency and reliability of the available optimisation procedures has been investigated for one of the tests described in Reference 17. A quadratic approximation method and corresponding optimisation program described in Reference 21 appeared to be satisfactory. This method is based on solving a sequence of auxiliary subproblems of quadratic programming. In these subproblems the Lagrangian associated with the primal problem is minimised and the Wolfe-Broyden-Davidon formula is employed to update the inverse of related Hessian matrix with each iteration. The objective function and all variables, as well as constraints, are scaled in the synthesis program. The algorithm applied in misation procedure stops when

$$\max\left(\|\text{grad } L^s(x)\|,\ |L^s(x) - F_o^s(x)|\right) \leqslant \varepsilon_1$$

and

$$\max\left(\max_{1 \leqslant j \leqslant m_i} g_j^s(x);\ \max_{1 \leqslant l \leqslant m_e} |h_l^s(x)|\right) \leqslant \varepsilon_2$$

where $L(x)$ is Lagrangian, $F_o(x)$ is the objective function, the upper index s indicates scaling and the constants ε_1 and ε_2 define the assumed accuracy level.

The program is written in FORTRAN IV and calculations were performed on a CDC CYBER-72 computer.

6.6 Results and analysis

All results presented in this Section are related to the solutions of Problems P_3 or P_4 satisfying Kuhn-Tucker stationarity conditions, and other stopping criteria applied in the optimisation procedure, with relatively high accuracy. Both the constants ε_1 and ε_2 used in these criteria have been assumed in calculations to equal 10^{-5} for any scaled problem.

The elements of the set $S1(P3)$ in Fig. 5 and results in Fig. 6 and 7 correspond to the solutions of Problem P_3

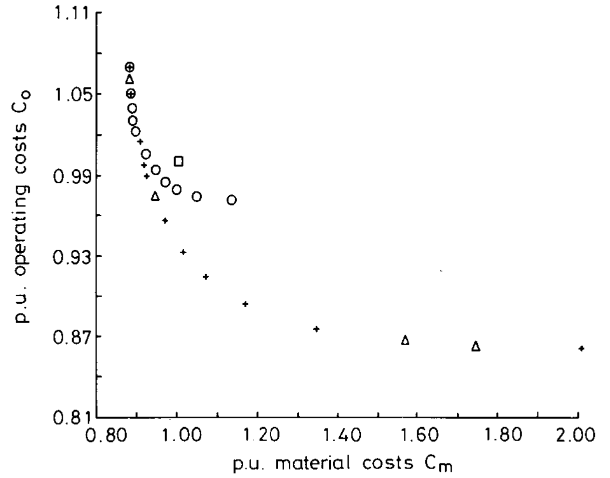


Fig. 5 *Estimation of compromise solutions set S in objective space for bicriterial optimisation*

Solutions of Problems P_3 and P_4
+ $S1(P3)$
○ $S2(P3)$
△ $S1(P4)$
□ IN DESIGN

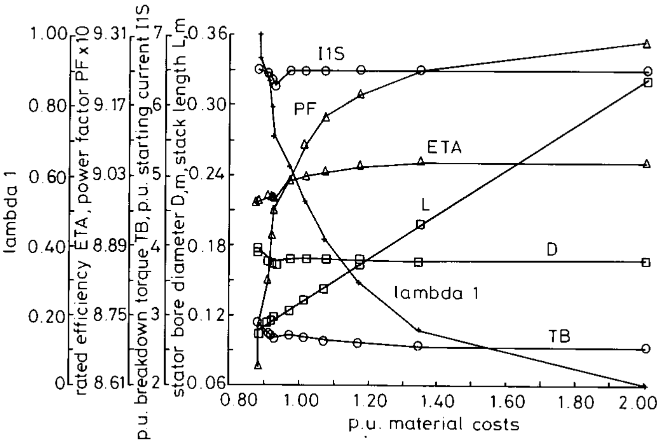


Fig. 6 *Influence of material costs C_m on design parameters and results associated with solutions of Problem P_3*

The set $S1(P3)$
Part 1
○ *I1S*
△ *TB*, *PF*, *ETA*
□ *L*, *D*
× lambda 1

for the values of the parameter λ_1, defined in eqn. 3 and represented by the marked points of the curve lambda1 in Fig. 6. Every point in Figs. 5, 6 and 7 can be referred to a design solution that, for an assumed value of the material costs C_m, realises locally minimal operating costs C_o. The values in Fig. 7 are $ETAL = \eta_l$, $PFL = \cos\phi_l$, $KMI = k_\mu$, $KE = k_E$ and BD = airgap magnetic-flux density B_δ. The values η_l, $\cos\phi_l$ are defined in eqn.

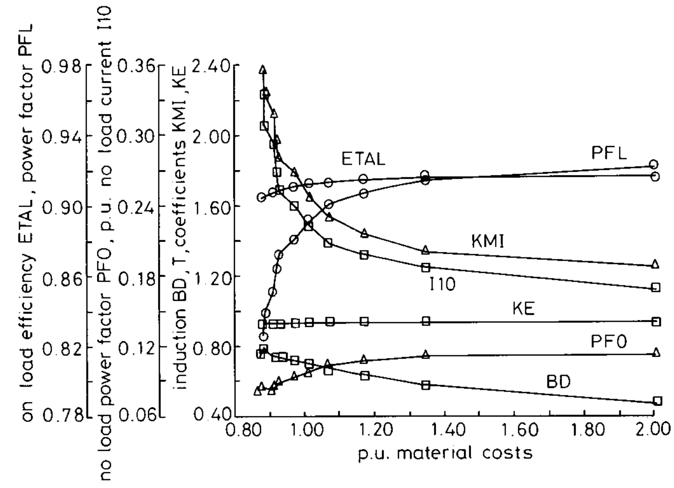


Fig. 7 *Influence of material costs C_m on design parameters and results associated with solutions of Problem P_3*

The set $S1(P3)$
Part 2
○ *ETAL*, *PFL*
△ *KMI*, *PFO*
□ *I10*, *KE*, *BD*

7, k_μ is the saturation factor of the magnetic circuit at full load and $k_E \equiv x_{13}$.

Fig. 5 represents a situation in the objective space and illustrates a relationship between p.u. operating costs C_o/C_o^o and p.u. material costs C_m/C_m^o, where the values $C_m^o = 3437.5$ kW/h and $C_o^o = 9041.6$ kW/h are reference values and correspond to the initial design solution as an example of design calculations in Reference 22. The point (C_m^o, C_o^o) is denoted IN DESIGN.

Computational experiments have been performed to explore the properties of the feasible region and criterial functions. The convexity was investigated over a set as line segments joining the different solutions of Problem P_3. The feasible region X and the criterial function C_o appeared to be not convex. A consequence of this situation is that two different sets, S_1 and S_2, of solutions of Problem P_3 defined by the same set $\{\lambda_1\}$, have been found.* If the points of the curve lambda1 in Fig. 6 represent the set $\{\lambda_1\}$, then both the sets S_1 and S_2 are represented in Fig. 5 by sets $S1(P3)$ and $S2(P3)$, respectively. In fact, sets S_1 and S_2 are continuous because of the assumed continuity of constraints and criterial functions.

Elements of the set denoted in Fig. 5 as $S1(P4)$ are solutions of Problem P_4 in the objective space determined for the criterial functions $f_1 = C_m$, $f_2 = C_o$ and the following values of the parameters $(j, k, \alpha_2) =$ (1, 2, 9600 kW/h), (1, 2, 8800 kW/h) and $(j, k, \alpha_1) =$ (2, 1, 5400 kW/h), (2, 1, 6000 kW/h). It was possible to obtain the elements of the sets $S1(P3)$ and $S1(P4)$ by starting from the same initial points, meaning that $S1(P4)$ is a subset of the set S_1.

If S_1 and S_2 are the only different sets of solutions of problem P_3, then an analysis is possible to decide whether an element of these sets belongs to the set S of the compromise solutions of Problem P_3 or not. There

* See Endnote.

exists almost always a level of uncertainty for the results of such an analysis because the number of solutions of very complicated problems must be limited. Applying the definition of the compromise solution set S in the objective space to the determined elements of sets S_1 and S_2 makes it possible to select a subset S^* that may belong to the set S. In the analysed case, the set S consists of points (C_m, C_o), defined by the design solutions, ensuring that operating costs C_o will be minimal, on the condition that the material costs are not higher than C_m. If $(C^2_{m5}/C^o_m, C^2_{o5}/C^o_o)$ denotes the fifth point of the set $S2(P3)$ from the left side of Fig. 5, then the solution of Problem P_4, determined for the parameters $(j, k, \alpha_2) = (1, 2, C^2_{o5})$, and a starting point being the third element of the set $S1(P3)$ from the left side of Fig. 5, is a value of the material costs C^1_m, where $C^1_m = 3120.7$ kW/h > 3084.9 kW/h $= C^2_{m5}$. This means that part of the set S_2 is superior to a subset of the set S_1 in the sense of the definition of the set S. Thus, the subset S^* comprises all elements of the set $S1(P3)$, the first five elements of the set $S2(P3)$ from the left side of Fig. 5 and all elements of the set $S1(P4)$. The first two points of the set $S1(P3)$ from the left side of Fig. 5 are the same as the corresponding points of the set $S2(P3)$.

The values of all optimisation variables corresponding to the assumed initial design (design I), as well as the designs with the minimal material costs (design A), and the minimal operating costs (design B) are given in Table 1.

Table 1: Optimisation variables for initial and optimal design solutions

Optimisation variables	Initial design solution (design I)	Optimal design solutions $F = C_m$ (design A)	$F = C_o$ (design B)	$F = C_o$ (design C)
x_1 m	0.185	0.17764	0.16780	0.19103
x_2 m	0.130	0.10651	0.32213	0.15470
x_3 m	0.0218	0.025585	0.018890	0.021948
x_4 m	0.0048	0.0052365	0.0045106	0.0052858
x_5 m	0.0037	0.0034	0.0034	0.0034
x_6 m	0.032	0.032634	0.029505	0.031691
x_7 m	0.0065	0.0066817	0.0066118	0.0081655
x_8 m	0.0155	0.011885	0.010461	0.010887
x_9 m	0.04	0.045268	0.040887	0.043947
x_{10} m	0.0005	0.00048406	0.00047328	0.00049859
x_{11} m	0.00125	0.001315	0.001315	0.001315
x_{12}	28.0	28.571	18.618	25.107
x_{13}	0.92466	0.92496	0.93852	0.93407

Designs B and C are two different design solutions obtained when minimising the operating costs C_o defined in eqn. 7. Design I corresponds with the point IN DESIGN in Fig. 5, designs A and B with the elements of the set $S1(P3)$ for $\lambda_1 = 1$ and $\lambda_1 = 0$, respectively, and design C with the element of the set $S2(P3)$ for $\lambda_1 = 0$.

Through the comparison of the results in Table 1 it is seen that a machine with minimal operating costs (design B) has a smaller inner stator diameter, smaller dimensions of the ring as well as stator and rotor slots, fewer turns in the stator slot and is much longer than the machine with minimal material costs (design A). The values of the stator bore diameter, the rotor tooth width and the number of turns in the stator slot are considerably higher, whereas the stator stack length is much smaller in design C than in design B.

The percentage costs of main construction materials are given in Table 2.

The cost of iron is a more significant part of the total material costs C_m than it is in the example given in Reference 9. An explanation of this may be that the copper/iron cost ratio assumed here is about 3 : 1 instead of 5.5 : 1 in Reference 9. The cost of iron and body in design B comprises 85% of the C_m. It is interesting to note that the percentage cost of copper in design B is reduced by more than twice in comparison with that in design A.

Table 2: Percentage material costs for optimal designs and design C

Material	Optimal design solutions Design A $F = C_m$)	Design B $(F = C_o)$	Design C $(F = C_o)$
copper	21.3%	9.7%	17.0%
aluminium	6.8%	5.3%	6.3%
iron	47.1%	69.2%	55.1%
body	24.8%	15.8%	21.6%

Full-load losses in various parts of the motors corresponding to designs A, B and C are summarised in Table 3. Additional full-load losses were assumed [24] to be constant and equalled 0.5% of the rated power P_N. Resulting efficiencies at full load differed relatively little from each other: $\eta^A_N = 89.7\%$ (design A), $\eta^B_N = 90.6\%$ (design B) and $\eta^C_N = 89.9\%$ (design C). The difference between efficiencies η^A_l and η^B_l at load $P_l = 0.75 P_N$ (which can be determined from Fig. 7) is about 1.3% and is higher than that between η^A_N and η^B_N. More significant is the difference between power factors at load P_l determined for designs A and B: $\cos\phi^A_l = 82.7\%$ and $\cos\phi^B_l = 92.2\%$, respectively. These are the main reasons why operating costs C^B_o are about 80% of C^A_o.

Table 3: Percentage full-load losses for optimal designs and design C

Components		Optimal design solutions Design A $(F = C_m)$	Design B $(F = C_o)$	Design C $(F = C_o)$
I^2R losses	Stator	51.0%	49.4%	47.7%
	Rotor	18.4%	23.5%	20.8%
Core losses	Total	19.7%	15.2%	20.5%
	Surface	0.7%	0.8%	0.8%
	Tooth pulsation	3.0%	3.8%	2.4%
Mechanical losses		6.5%	7.1%	6.6%

In all three designs in Table 3 the percentage I^2R losses are nearly the same (comprising about 70% of the total loss) but their distribution between the stator and rotor differs. In both designs B and C the stator I^2R loss is reduced in comparison with that in design A, whereas the rotor I^2R loss is higher. The core losses are reduced only in design B; in design C they have even increased. The reduction of the core losses in design B in comparison with those in design A is related to the corresponding change of the no-load stator current $I10$ and no-load power factor PFO presented in Fig. 7.

Some results of magnetic calculations related to all optimal design solutions analysed in this Section are presented in Table 4. These design solutions correspond with the elements of the estimation S^* of the compromise set S.

Table 4 shows that despite any direct restriction imposed on magnetic induction values when defining the

Table 4: Extreme values of magnetic flux densities

	Minimal induction T	Maximal induction T
Airgap	0.469	0.755 (0.734)
Stator teeth	1.181	1.694
Rotor teeth	1.012	1.679 (1.668)
Stator yoke	0.661	1.988
Rotor yoke	0.874	1.591

feasible region, these values have remained at reasonable intervals. In general, the minimal values are associated with the machine most expensive for a manufacturer and the values in the second column with the cheapest. The magnetic inductions in the airgap and in the rotor teeth attain their maximum for the machine not optimal from a manufacturer's viewpoint (see curve BD in Fig. 7). The values of these inductions for design A are given in the brackets.

An active constraints set alters according to the choice of the parameters (λ_k) in eqn. 3 or (f_j, α_k) in eqn. 5. For example, if the objective function F in eqn. 3 is assumed to be operating costs C_o, then $\lambda_1 = 0$, $\lambda_2 = 1$, $F = C_o$ and the following ten constraints are ative in both stationary points: g_1, g_2, g_7, g_8, g_{10}, $g_{11}^* = x_9/x_6 \leqslant 1.4$, $g_{12}^* = d_{in} \leqslant 0.0014$, where d_{in} is the insulated stator wire diameter, $g_{13}^* = r_{22} \geqslant 0.00125$, where r_{22} is the radius of the rotor slot bottom, $g_{14}^* = x_5 - d_{in} \geqslant 0.002$, and h_1. The meaning of the asterisk is the same as in Section 6.4. The constraints g_1, g_7, g_8, g_{14} were active for all solutions of Problems P_3 and P_4.

The maximal calculation time for one satisfactory optimisation run was about 600 cpu seconds, which corresponds to about 200 gradient calls.

7 Remarks and conclusions

A bicriterial optimisation problem related to induction-motor designing has been defined and solved.

Applying the quadratic approximation method appeared to be satisfactory for finding all the desired solutions of corresponding scalar optimisation Problems P_3 or P_4. An assumption about a relatively high accuracy of optimisation calculations was necessary to obtain correct results in the described multicriterial approach.

It has been found that two different sets of solutions of Problem P_3, as well as P_4, satisfy the Kuhn-Tucker stationarity conditions. In this paper they are defined in the objective space as S_1 and S_2. It is difficult and often impossible to prove, in a strict mathematical sense, a general property of a high nonlinear, complicated mathematical model defined in multidimensional space. Thus, one cannot exclude an existence of other sets beyond the above solutions, but, according to the number of performed calculations, such results are not very likely in the realm of practical interest. A comparison of the values of the optimisation variables and other results concerning designs B and C defined in Table 1, as well as both the sets $S1(P3)$ and $S2(P3)$ presented in Fig. 5, may be helpful when searching for an optimal design solution of a similar induction motor.

The structure of the compromise solution set S, being partly determined in Section 6.6, involves a discontinuity of the corresponding compromise set P in the decision space. This conclusion seems to be important in the case of induction-motors series designing.

It is worth pointing out that if the parameter λ_k in the objective function (eqn. 3) associated with operating costs C_o is relatively large, then one can expect two different solutions for Problem P_3. When the value of this parameter is larger, the distance between obtained minima is greater in both the objective and decision spaces. The very likely number of solutions of Problem P_4 is one or two and depends on the parameters α_1 or α_2, as can be seen in Fig. 5.

A conclusion arising from the results presented in Fig. 5 is that the multipliers method cannot be used to determine the global compromise solution set without an additional analysis. Indeed, the local compromise sets $S1(P3)$ and $S2(P3)$ were determined in the example by means of the multipliers method, but the final decision to select the elements of the estimate S^* of the compromise set S could be made after applying the definition of the set S. Employing the PEC [13] or PIC methods [14] is an alternative and more general approach to find the set S.

Results in Table 1 show that a consequence of a reduction of operating costs C_o is not only increasing the stator and rotor stack lengths but also decreasing the cross-section area of the stator slot. This relationship appears to be true for almost all permissible values of the parameter λ_1 defining the objective function (eqn. 3). Because iron costs are a predominant part of the material costs (see Table 2) the above property has only a small influence on curve L in Fig. 6.

It is well known that an induction motor with lower operating costs has higher efficiencies and power factors at any load including the no-load operation, and lower magnetic flux densities, currents, saturation factor and maximal torque. Results in Figs. 6 and 7 show that the above relationships are not always true.

An assumption about the values of magnetising parameters has been accepted in the paper. It reduces computational effort and complexity of the synthesis program but increases the inaccuracy of the results. This remark seems to be important because the value of the operating costs concerns a load different from the rated one, and the constraint g_1, dependent on the starting condition of the motor, is always active. A necessary analysis will be performed in the future.

The reason behind assuming that some design parameters are independent variables is to perform a qualitative rather than a quantitative analysis, but it enables the designer to draw some valuable conclusions. In the analysed case, for instance, the stator slot opening, as well as airgap length, should be as small as possible, and the stator slot fill factor should be as high as possible, for all compromise design solutions.

It would be rather difficult to get from the results presented in Figs. 5, 6 and 7 such smooth curves as one could expect taking into account some earlier works. A careful investigation shows that irregularities of these curves result not from inaccuracies of optimisation calculations but from properties of the assumed feasible region and objective function.

The expenditure of work and time to obtain the results presented in this paper is considerably large for both the designer and computer. Beyond this, it must be taken into account that further calculations are usually necessary. They are related to

(i) the fact that the set Y, comprising discrete variables vectors y, consists of more than one element

(ii) the restoration of a desired discrete form for some continuous variables

(iii) the analysis of errors originated in the assumed database, motor model, optimisation procedure and the decision process of the final selection of one design solution

(iv) the analysis of design solutions in the vicinity of the optimal point

(v) the unification of some parameters in the case of motors series designing

(vi) the more accurate analysis concerning strength, reliability, noise and other calculations usually not performed at the first stage of optimisation.

If the set Y of discrete variables consists of more than one element, then defining for these elements all subsets S^*, described in Section 6.6, and applying the theory presented, for instance in References 12, 13 and 14, seems to be a reasonable approach to determine an interesting part of the set S of compromise solutions of Problem P_2. In this approach the determination of both the compromise set P in the decision space and the set S in the objective space is a simultaneous process.

A major purpose of this paper is to investigate and present possible benefits arising from the application of the multicriterial approach to induction-motor design. It is seen in Fig. 5 that an assumption about slightly higher operating costs than the minimal ones reduces the material costs considerably. A similar remark occurs for the machine with the minimal material costs. From the results presented in Figs. 5, 6 and 7, it is an easy task to obtain information concerning the consumer, as well as producer optimised motor. The former machine is described by the value lambda1 = 0.5 and the latter by the value lambda1 = 1. It is seen in Fig. 5 that only a fraction of the whole compromise set S and related information is interesting for a designer in the case of single-motor design. In the case of the motors series the other design solutions defined by the values lambda1 < 0.5 may be interesting. Thus, the multicriterial approach appears to be more general than those described in the majority of earlier works dealing with induction-motor design. Results of this approach enable a designer to make a proper decision related to the selection of the best design solution with higher probability than in the case of scalar optimisation.

It seems to be justified to draw a conclusion from presented results that the applied procedure of designing is valid, and both the computational model and synthesis program briefly described in this paper can be used, after suitable modifications, to perform design optimisation calculations for other types of induction motors.

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9 Endnote

Readers should be warned that standard cubic spline functions were used in this paper to approximate some experimental and complicated analytical curves. If they were replaced, however, by the more sophisticated spline functions then both the criterial functions C_m and C_o appeared to be convex, and the set S_1 was the only set of solutions of Problem P_3 or P_4.

See author's subsequent papers:

'Influence of material cost on design parameters and properties of optimum designed squirrel-cage induction motor'. Proceedings of ICEM, Pisa, Italy, 1988, pp. 417–421

'Discrete multicriterial optimisation of squirrel-cage induction motor design' (in preparation)

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